2018 International Conference on Computational Science and Engineering (ICCSE 2018)

Investigation of a New Fractional-order System

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Keywords: Dynamics, Fractional-order system, Chaos.

Abstract: In this paper, the dynamics of a new fractional-order system is investigated. Firstly, the numerical solutions of the system by the improved predictor-corrector algorithm are obtained. Based on this, the dynamics of the system is analyzed by numerical simulations. A chaotic attractor and periodic orbits with different values of the derivative order or the system parameter are demonstrated.

1. Introduction

Recently, researchers proposed many typical new fractional-order systems. Meanwhile, dynamics of these systems were studied in details including all kinds of periodic solutions, chaos windows, bifurcations, and boundary and interior crises. A fractional-order Chua system with a derivative order 2.7 exists chaos [1]. A Duffing system with derivative order less than 2 can behave in a chaotic manner [2]. In [3], the authors numerically investigated the chaotic behaviors for the fractional-order Chen system with the lowest order 0.3, which was the lowest-order chaotic system among all the found chaotic systems to date. Chaos and bifurcation were investigated for the fractional-order T system with the complex variables [4]. In [5], the boundary and interior crises were found in a fractional-order Duffing system.

2. A new fractional-order system

In this section, a new fractional-order system is proposed, which can be described by the following fractional differential equations,

$$\begin{cases} D^{q_1}x = ax - y \\ D^{q_2}y = x - z - by \\ D^{q_3}z = ax + 4z(y-2) \end{cases} \tag{1}$$

where x, y, z are the state variables, and a, b the system parameters.

When the derivative orders are taken as $q_1 = q_2 = q_3 = 0.99$, the parameters $a = 0.3, b = 0.02$, and the initial conditions $(x_0, y_0, z_0) = (-0.5, -1, 1)$, the system (1) is chaotic, and the attractor with a one-scroll is depicted in the Fig.1.

  DOI: 10.25236/iccse.18.024

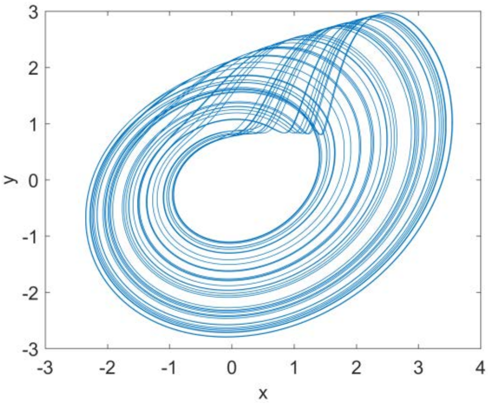


Fig.1. The chaotic attractor of the system (11) projected onto $x-y$ plane

Using the improved predictor-corrector algorithm, numerical solutions of the system (1) are given as follows

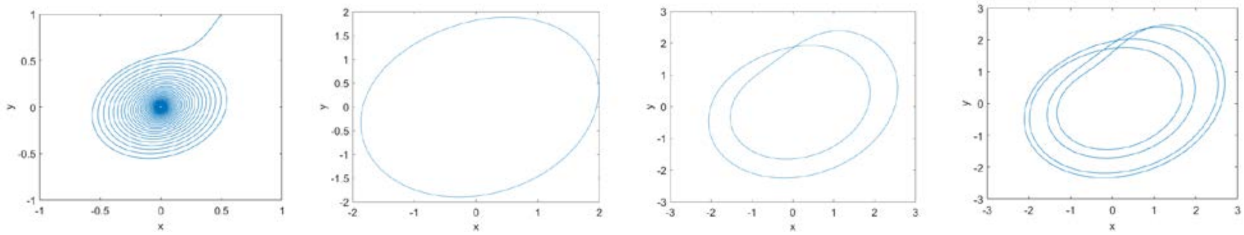
$$\begin{cases} x_{n+1} = x_0 + \dfrac{h^{q_1}}{\Gamma(q_1+2)}\{ax_{n+1}^p - y_{n+1}^p\} + \sum\limits_{j=0}^{n} \alpha_{1,j,n+1}(ax_j - y_j) \\ y_{n+1} = y_0 + \dfrac{h^{q_2}}{\Gamma(q_2+2)}\{x_{n+1}^p - z_{n+1}^p - by_{n+1}^p\} + \sum\limits_{j=0}^{n} \alpha_{2,j,n+1}(x_j - z_j - by_j) \\ z_{n+1} = z_0 + \dfrac{h^{q_3}}{\Gamma(q_3+2)}\{ax_{n+1}^p + 4z_{n+1}^p(y_{n+1}^p - 2)\} + \sum\limits_{j=0}^{n} \alpha_{3,j,n+1}(ax_j + 4z_j(y_j - 2)) \end{cases}, \quad (2)$$

where

$$\begin{cases} x_{n+1}^p = x_0 + \dfrac{h^{q_1}}{\Gamma(q_1)} \sum\limits_{j=0}^{n} \beta_{1,j,n+1}((ax_j - y_j)) \\ y_{n+1}^p = y_0 + \dfrac{h^{q_2}}{\Gamma(q_2)} \sum\limits_{j=0}^{n} \beta_{2,j,n+1}(x_j - z_j - by_j) \\ z_{n+1}^p = z_0 + \dfrac{h^{q_3}}{\Gamma(q_3)} \sum\limits_{j=0}^{n} \beta_{3,j,n+1}(ax_j + 4z_j(y_j - 2)) \end{cases}, \quad (3)$$

$$\begin{cases} \alpha_{i,j,n+1} = \begin{cases} n^{q_i+1} - (n-q_i)(n+1)^{q_i},\ j=0 \\ (n-j+2)^{q_i+1} + (n-j)^{q_i+1} - 2(n-j+1)^{q_i+1}, 1 \le j \le n \\ 1,\ j = n+1 \end{cases} \\ \beta_{i,j,n+1} = \dfrac{h^{q_i}}{q_i}((n-j+1)^{q_i} - (n-j)^{q_i}), 1 \le j \le n \end{cases}. \quad (4)$$

Compared with an integer-order system, the derivative order is an important parameter for a fractional-order system. For the system (1), the system parameters and the initial conditions are fixed. The phase diagrams with different values of the derivative order q are employed to demonstrate the behavior of the system (1), see Fig.2. From which it can be seen that the system (11) converges to a fixed point when $q = 0.90$, is period-1 for $q = 0.92$, period-2 for $q = 0.95$, and period-4 for $q = 0.955$.

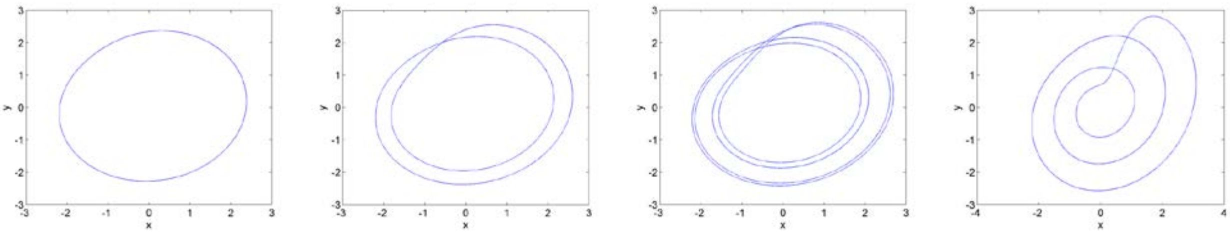


(a) (b) (c) (d)

Fig.2. The attractor of the system (11) projected onto $x-y$ plane (a) $q=0.90$; (b) $q=0.92$; (c) for $q=0.95$; (d) $q=0.955$.

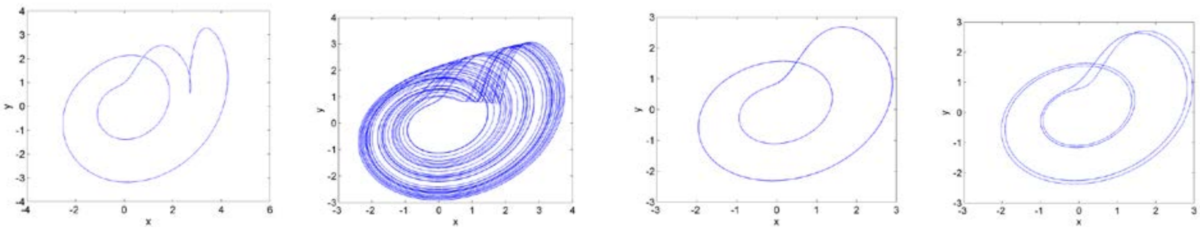
When the derivative order q is taken as 0.99 and the other parameters are fixed, the dynamics of the system (11) with the variation of the parameter a is studied. The phase diagrams with the different values of the parameter are used to show the behavior of the system (1), see Fig.3. It can be seen that the system is period-1 for $a=0.1$, period-2 for $a=0.15$, period-4 for $a=0.17$, and period-3 for $a=0.25$. Clearly, the evolution of the period-doubling scenario can be observed easily.

Meanwhile, the dynamics of the system (11) with the variation of the parameter b is studied when $q=0.99$ and $a=0.3$. From Fig.4, it can be seen that the system is period-3 for $b=-0.05$, chaotic for $b=0$, period-4 for $b=0.035$, and period-2 for $b=0.04$.



(a) (b) (c) (d)

Fig.3. The attractor of system (11) projected onto $x-y$ plane (a) $a=0.10$; (b) $a=0.15$; (c) $a=0.17$; (d) $a=0.25$.



(a) (b) (c) (d)

Fig.4. The attractor of system (11) projected onto $x-y$ plane (a) $b=-0.05$; (b) $b=0$; (c) $b=0.035$; (d) $b=0.04$.

3. Summary

In this paper, a new system with fractional order with a one-scroll chaotic attractor is proposed. Firstly, the numerical solutions of the system by the improved predictor-corrector algorithm are obtained. The dynamics of the system, including chaotic attractor and periodic orbits, is analyzed by numerical simulations.

Acknowledgements

This research was financially supported by the National Science Foundation of China (NSFC) under the grant No. 11702194, and the National Science Foundation of Gansu Provience under the grant No. 1610RJZE138.

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